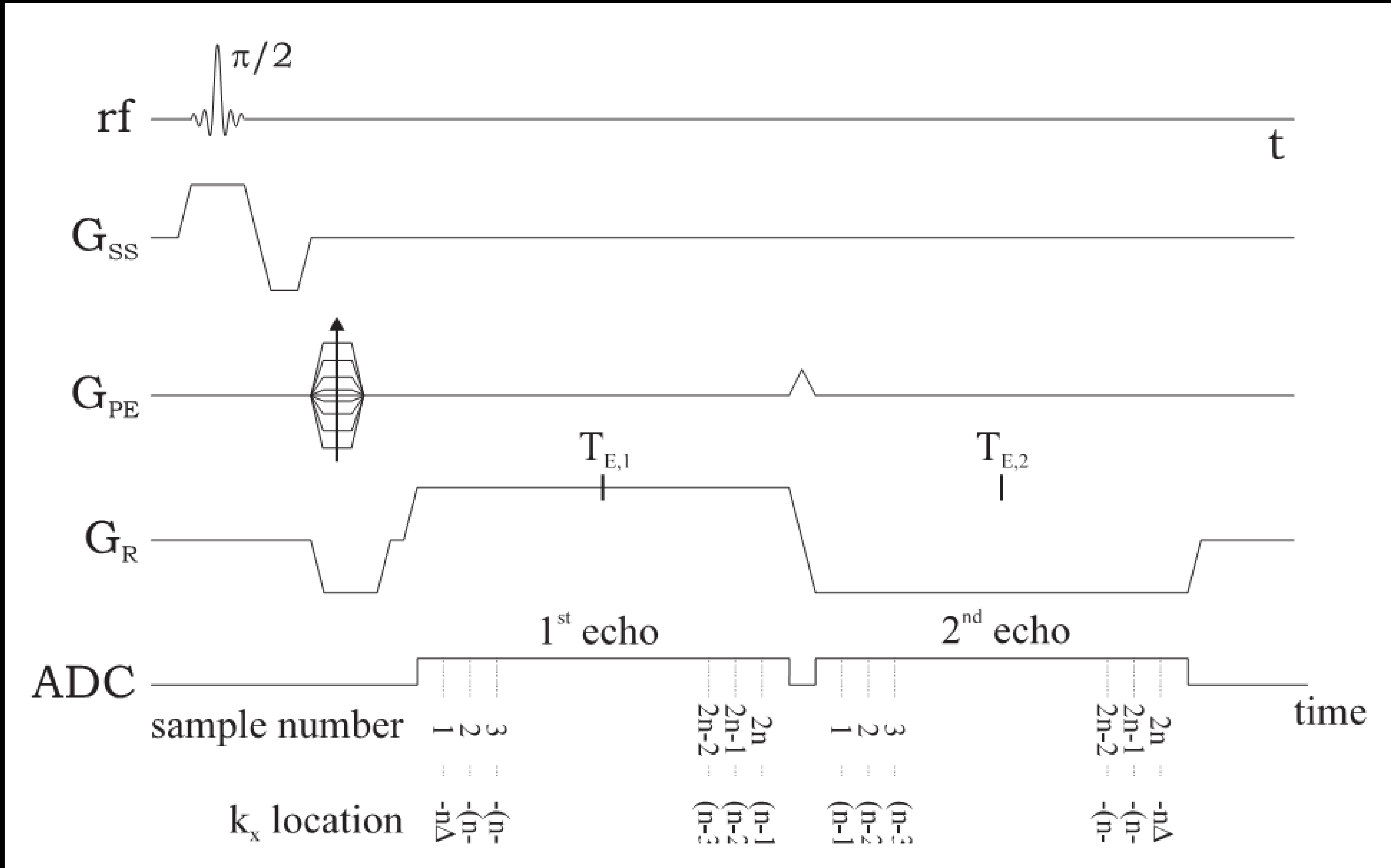
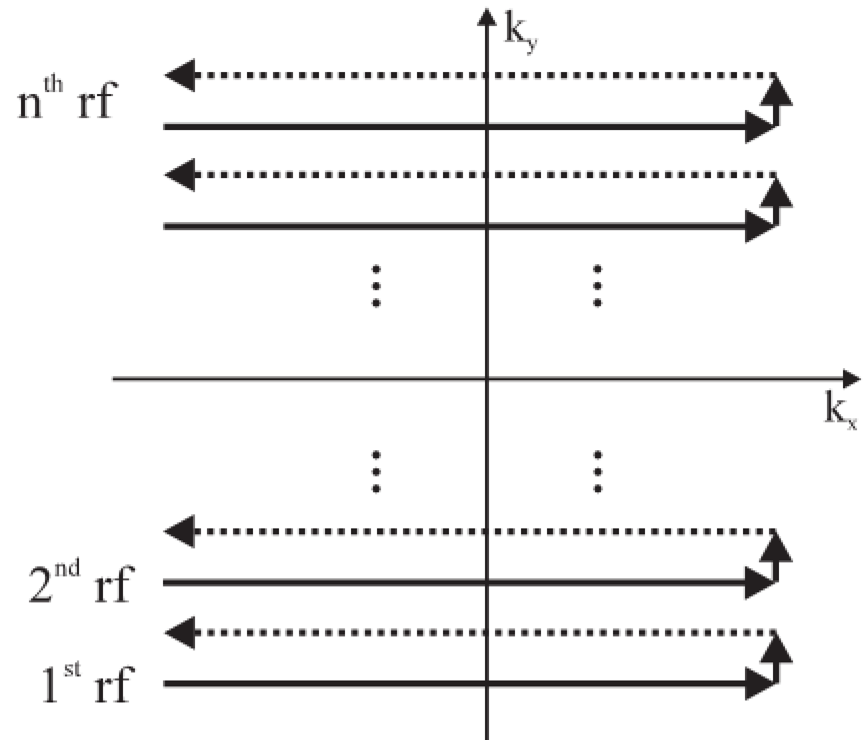
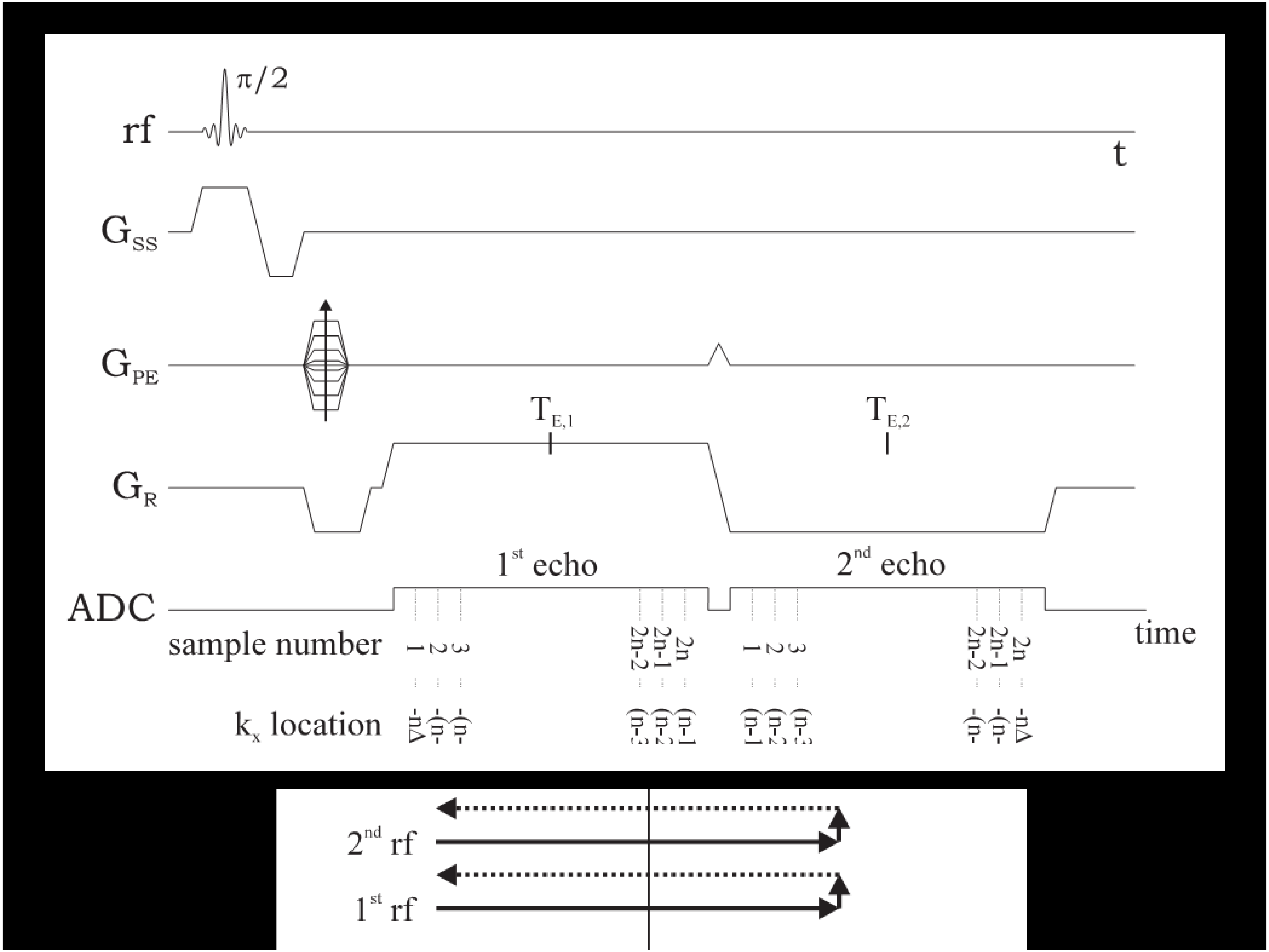
Chapter 19 and 22



π/2
rf
t
G_SS
G_PE
G_R
T_E,1
T_E,2
1st echo
2nd echo
ADC
sample number
time
k_x location

segmented *k*-space image (double echo example)





π/2
rf
t
G_SS
G_PE
T_E,1
T_E,2
G_R
1st echo
2nd echo
ADC
time
sample number
1
2
3
2n-2
2n-1
2n
k_x location
-nΔ
2nd rf
1st rf

Even Lines

$$u_{first\ echo}(k_x, k_y) =$$

$$\Delta k_x \Delta k_y \sum_{r=-n}^{n-1} \sum_{p=-n/2}^{n/2-1} e^{-T_{E,1}/T_2^*} e^{-r\Delta t/T_2^*} \delta(k_x - r\Delta k_x)\delta(k_y - 2p\Delta k_y)$$

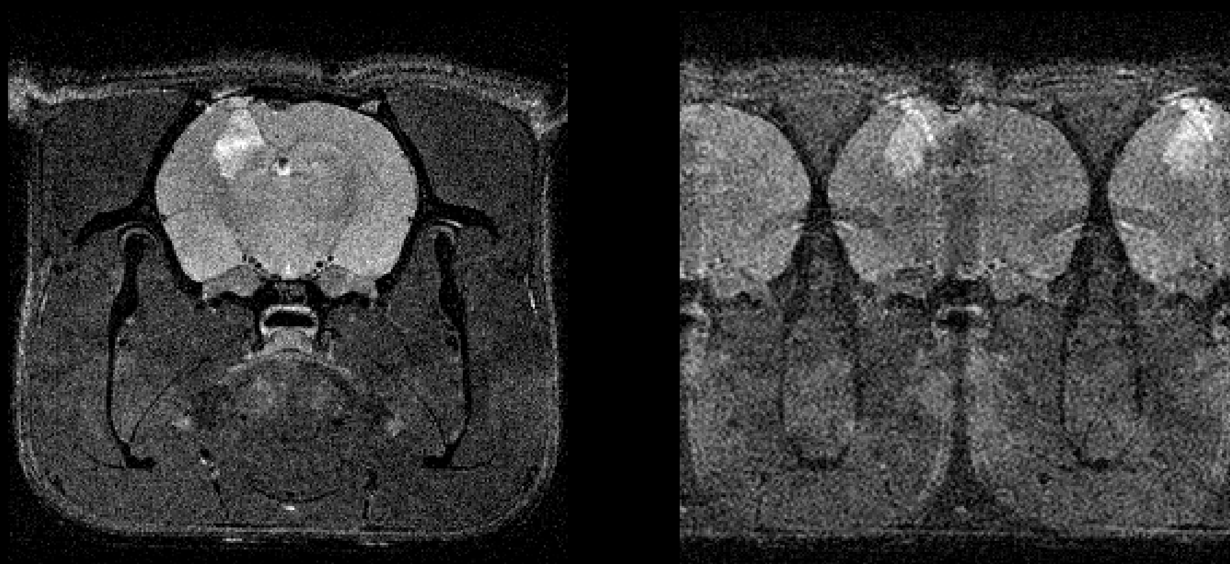
Odd Lines

$$u_{second\ echo}(k_x, k_y) =$$

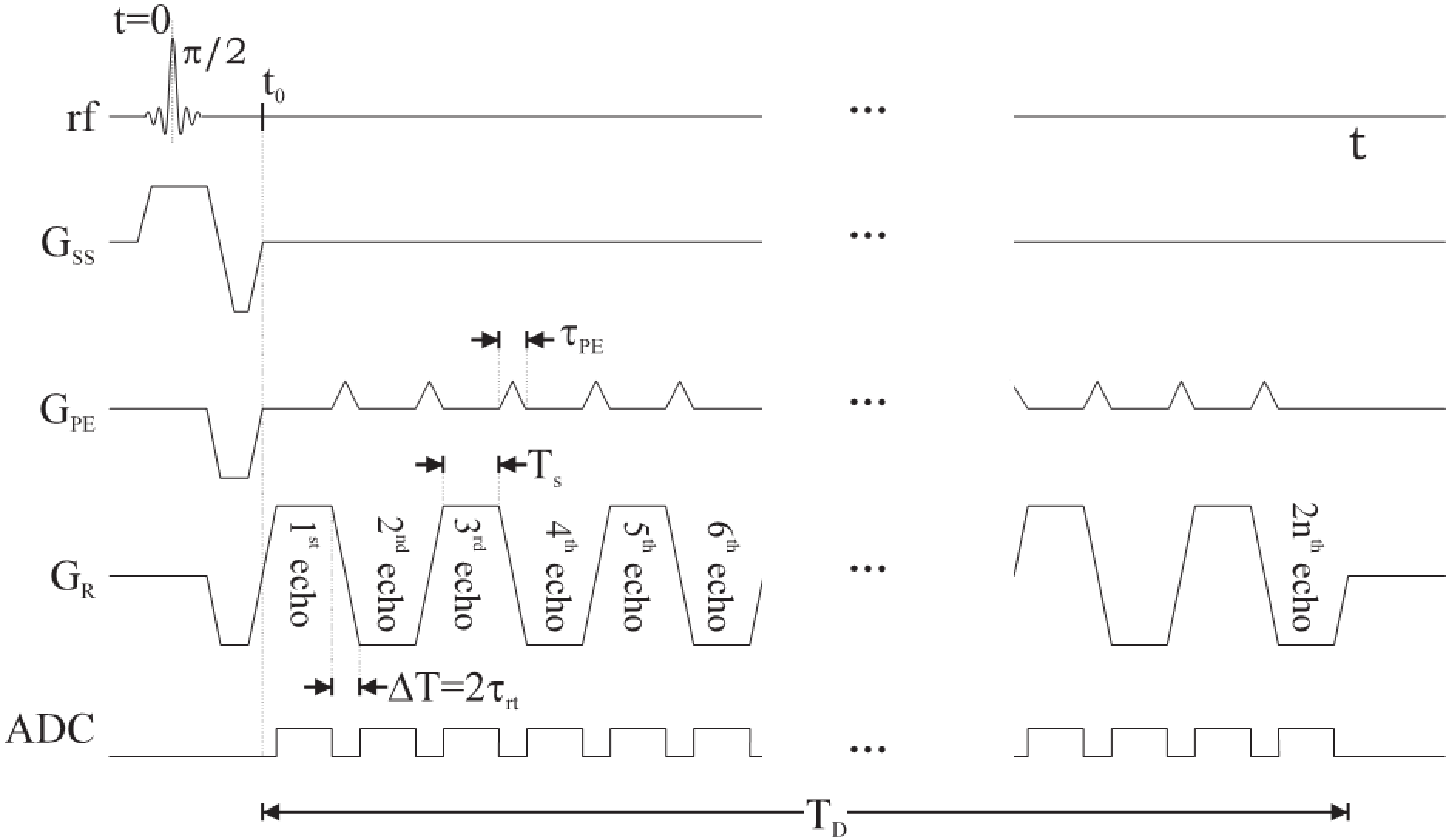
$$\Delta k_x \Delta k_y \sum_{r=-n}^{n-1} \sum_{p=-n/2}^{n/2-1} e^{-T_{E,2}/T_2^*} e^{+r\Delta t/T_2^*} \delta(k_x - r\Delta k_x)\delta(k_y - (2p+1)\Delta k_y)$$

Combined Sampling Function

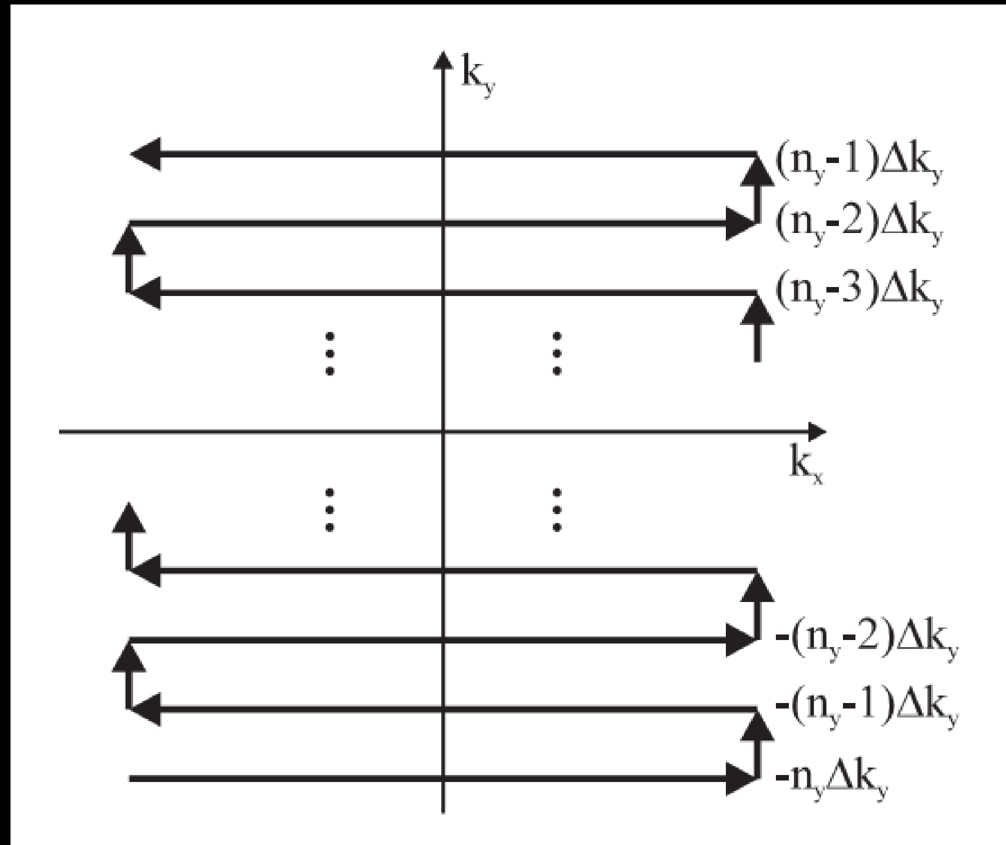
$$u(k_x, k_y) = u_{first\ echo}(k_x, k_y) + u_{second\ echo}(k_x, k_y)$$



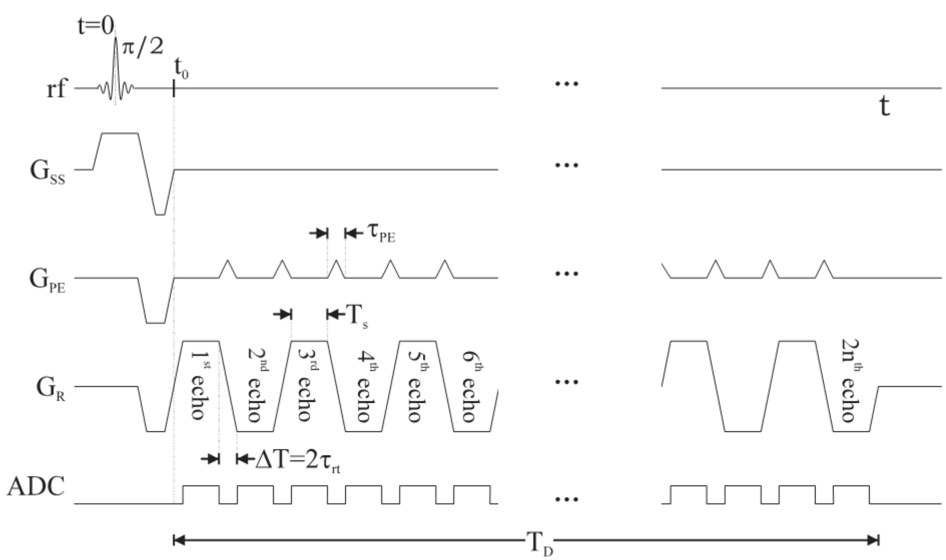
EPI sequence with rectilinear sampling and sequential coverage of k-space



TE
TR
Lx
Ly
Δx
Δy



Time at which the central part of the k-space is sampled is the echo time



TD – Total time for which the read out gradient is played out

T_S – total sampling time for an echo

ΔT – peak-to-peak gradient ramp time

τ_{rt} – gradient ramp time from zero to peak

Δt – sampling time interval in read-out direction

τ_{pe} – duration of the phase-encode gradient blip

ΔG_{yo} – phase-encode gradient peak

$$\begin{aligned} T_D &= N_y(T_s + \Delta T) \\ &= N_x N_y \Delta t + N_y \Delta T \end{aligned}$$

$$T_T = \tau_{rf}/2 + t_0 + T_D$$

$$T_{E,\vec{k}=0} = \frac{N_y + 1}{2} N_x \Delta t + (N_y + 1)\tau_{rt} + t_0$$

Setting the Imaging Parameters

Lx, Gx, Δx, Nx
Ly, ΔGyo, Δx, Ny

You could fix – Gx to Gx,max to minimize scan time and T2* filtering effects.

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$$\Delta t = \frac{1}{\gamma G_x L_x}$$ $$N_x = \frac{L_x}{\Delta x}$$

$$\Delta G_{y_0} = 2/(\gamma L_y \tau_{pe})$$

$$\int_t^{t+\tau_{pe}} dt'\, \Delta G_y(t') = \frac{1}{\gamma L_y}$$

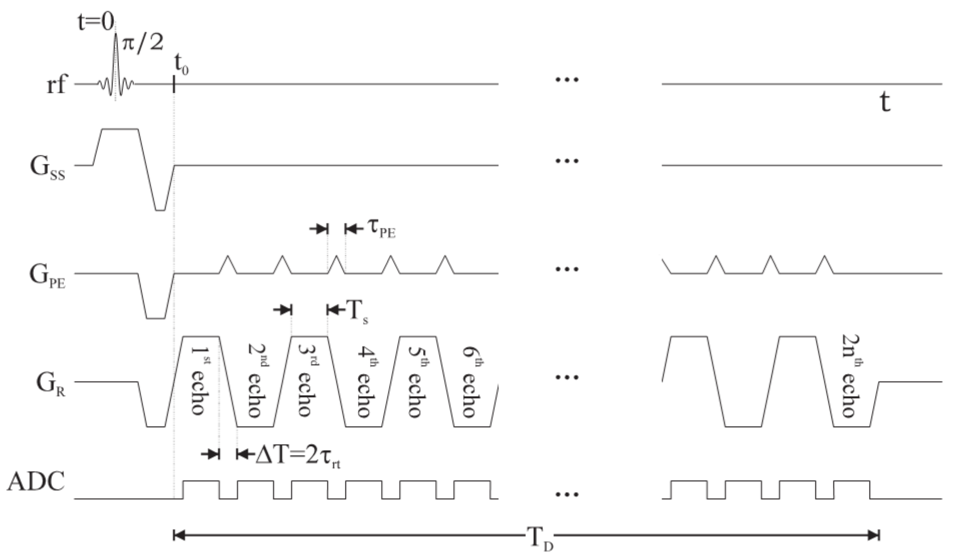
$$\tau_{pe} = 2\tau_{rt}$$

$$\Delta G_{y_0} = 1/(\gamma L_y \tau_{rt})$$

# of PE lines that could be sampled is determined by a few parameters:
T2* filtering effects, Signal available and maximum gradient strength or sampling time for a given read-out.

$$N_{y,max} \simeq \frac{T_2^*}{N_x \Delta t + 2\tau_{rt}}$$ $$\Delta y_{min} \simeq \frac{L_y}{N_{y,max}}$$

Effect of Chemical-shift (Distortion) in phase encoding direction in EPI (Rectilinear sampling)



Δt is DW

$$\Delta\phi_r = 2\pi\gamma[G_r x\mathrm{DW} + \Delta B_0(x, y, z)\mathrm{DW}],$$

$$\Delta\phi_{\mathrm{pe}} = 2\pi\gamma\Delta G_{\mathrm{pe}} y \tau_{\mathrm{pe}}$$

Conventional GRE

EPI - GRE

$$\Delta\phi_r = 2\pi\gamma[G_r x\mathrm{DW} + \Delta B_0(x, y, z)\mathrm{DW}], \tag{4}$$

$$\Delta\phi_{\mathrm{pe}} = 2\pi\gamma[G_{\mathrm{pe}} y \tau_{\mathrm{ramp}} + \Delta B_0(x, y, z)(2\tau_{\mathrm{ramp}} + N.\mathrm{DW})]$$

Effect of Chemical-shift (Distortion) in phase encoding direction in EPI (Rectilinear sampling)

$$\phi_\sigma(k_x,(p+1)\Delta k_y) - \phi_\sigma(k_x,p\Delta k_y) = \gamma \Delta B_\sigma \Delta\tau_{pe}$$

$$\Delta\tau_{pe} \equiv T_s + 2\tau_{rt}$$

$$\begin{aligned} n_{\sigma,pe} &= \frac{N_y(\phi_\sigma(k_x,(p+1)\Delta k_y) - \phi_\sigma(k_x,p\Delta k_y))}{2\pi} \\ &= N_y \gamma \Delta B_\sigma \Delta\tau_{pe} \end{aligned}$$

Along **Read** direction…

$$n_{\sigma,r} = N_x \gamma \Delta B_\sigma \Delta t$$

Comparing both…

$$\frac{n_{\sigma,pe}}{n_{\sigma,r}} = \frac{N_y}{N_x} \cdot \frac{\Delta\tau_{pe}}{\Delta t}$$

Ch22 - Quantifying Basic Tissue Properties of Spindensity, T1 and T2

$$M_0 \simeq \rho_0 \frac{s(s+1)\gamma^2\hbar^2}{3kT} B_0 \qquad (\hbar\omega_0 \ll kT)$$

$$\hat{\rho}(T_R, T_E) = \rho_0 e^{-T_E/T_2}\left(1 - e^{-T_R/T_1}\right)$$

$$T_R \gg T_E$$

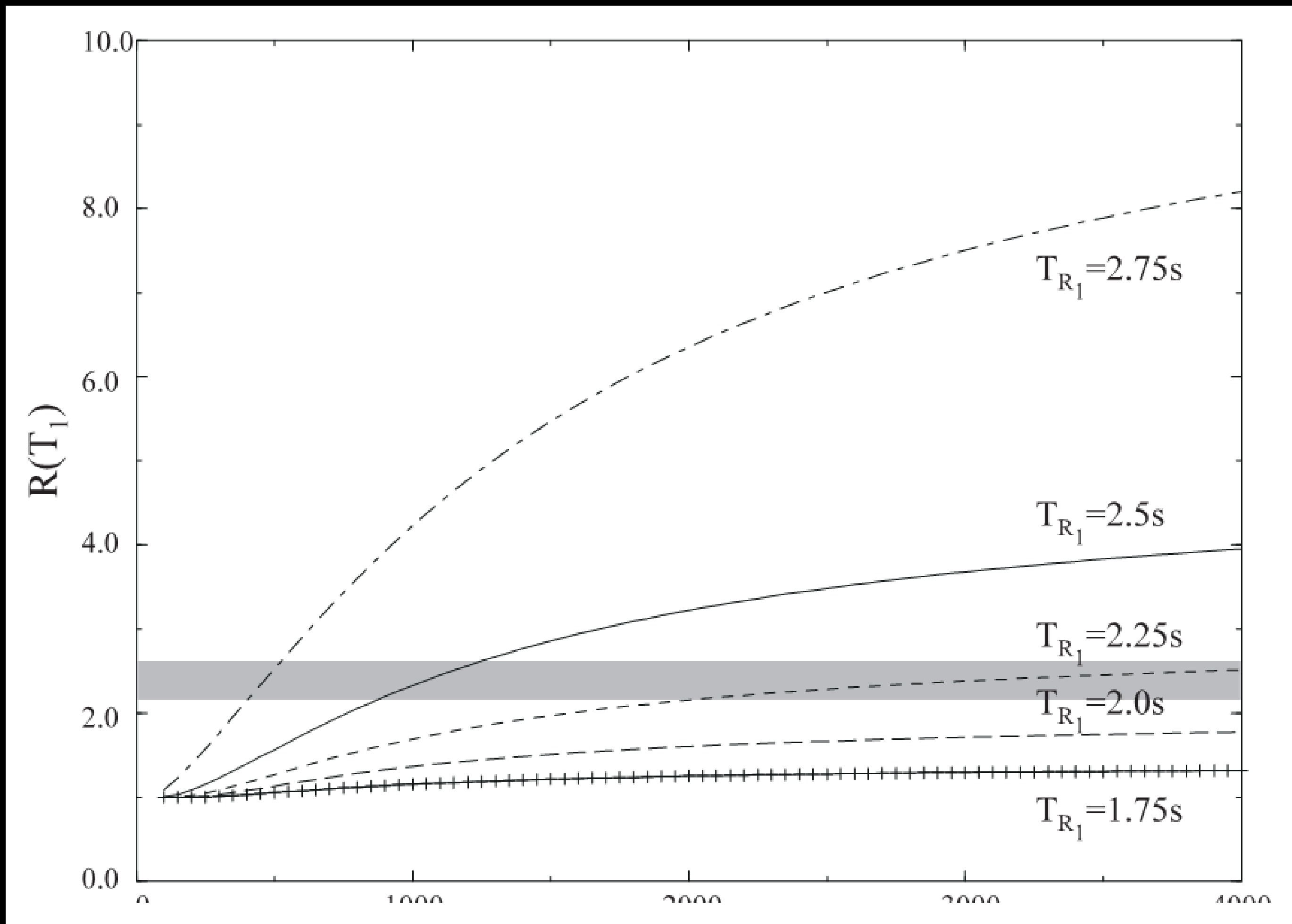
$$T_E \simeq T_2$$

$$T_E \ll T_2$$

$$T_R \gg T_1$$

$$T_R \ll T_1$$

$$R \equiv \frac{\rho_1}{\rho_2} = \frac{(1 - e^{-T_{R_1}/T_1})}{(1 - e^{-T_{R_2}/T_1})}$$

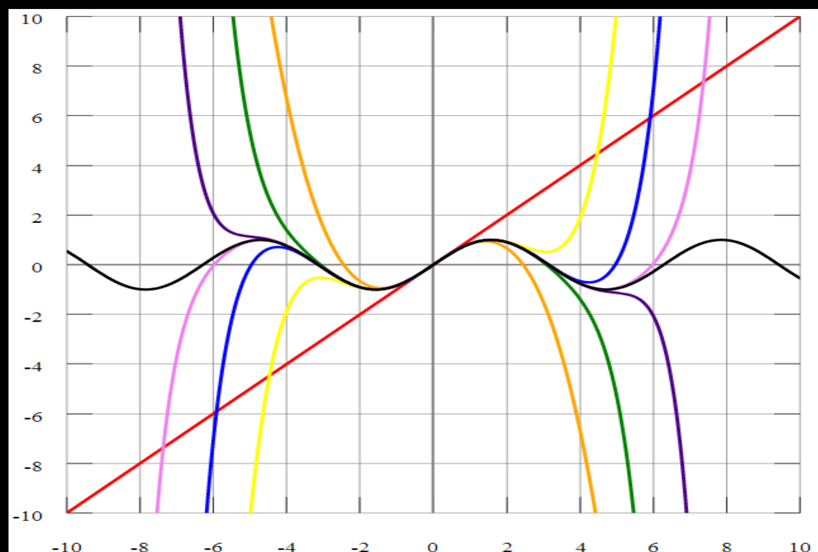


Ch22 – Error Propagation
Taylor Expansion

X= f(u,v)

If we have a small error in the independent variables *u* or *v* that we are essentially measuring, then what is the resultant error in the dependent variable *X* that we are attempting to quantify

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f^{(3)}(a)}{3!}(x-a)^3+\cdots.$$



As the degree of the Taylor polynomial rises, it approaches the correct function. This image shows   and Taylor ***Sin(x)*** approximations, polynomials of degree 1, 3, 5, 7,9, 11 and 13.

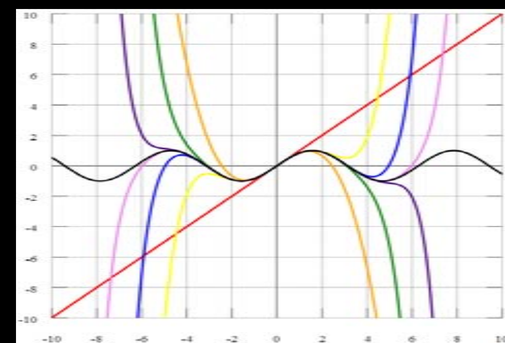
Wikipedia.org

Ch22 – Error Propagation
Taylor Expansion

Essentially Taylor Series expansion of a function to the first order, around a particular value of the independent variable, computes the linear approximation of the function around that value.

1D function

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f^{(3)}(a)}{3!}(x-a)^3+\cdots.$$



F(x) = some differentiable function in x

So, if we deviate from 'a' by a small amount, what will the value of *f(x)* be can be obtained in the first order (as long as (x-a) is really small), from the first order term of the Taylor series expansion of *f(x)*.

2 D function

$$\begin{aligned} f(x,y) &\approx f(a,b)+(x-a)\,f_x(a,b)+(y-b)\,f_y(a,b)\\ &\quad+\frac{1}{2!}\left[(x-a)^2\,f_{xx}(a,b)+2(x-a)(y-b)\,f_{xy}(a,b)+(y-b)^2\,f_{yy}(a,b)\right], \end{aligned}$$

Wikipedia.org

Ch22 – Error Propagation
Taylor Expansion

Example $V_0 = L_0 W_0 H_0$

Ignoring the higher order terms

$$V \simeq V_0 + \Delta L \left(\frac{\partial V}{\partial L}\right)_{W_0 H_0} + \Delta W \left(\frac{\partial V}{\partial W}\right)_{L_0 H_0} + \Delta H \left(\frac{\partial V}{\partial H}\right)_{L_0 W_0}$$

Generalization

$$x = f(u, v, \ldots)$$

$$\sigma_x^2 \simeq \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2 + \cdots + 2\sigma_{uv}^2 \left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right) + \cdots$$

If noise in u and v are uncorrelated then σ_{uv}^2 is 0, so

$$\sigma_x^2 \simeq \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2 + \cdots$$

Bevington, Philip R.; Robinson, D. Keith (2002), Data Reduction and Error Analysis for the Physical Sciences (3rd ed.), McGraw-Hill,

$$\hat{R} \equiv \frac{\hat{\rho}_1}{\hat{\rho}_2} \simeq R\left(1 + \frac{\epsilon_1}{\rho_1} - \frac{\epsilon_2}{\rho_2}\right)$$

$$\frac{\sigma_{\hat{R}}^2}{R^2} = \sigma^2\left(\frac{1}{\rho_1^2} + \frac{1}{\rho_2^2}\right) = \frac{1}{\mathrm{SNR}_1^2}(1 + R^2)$$

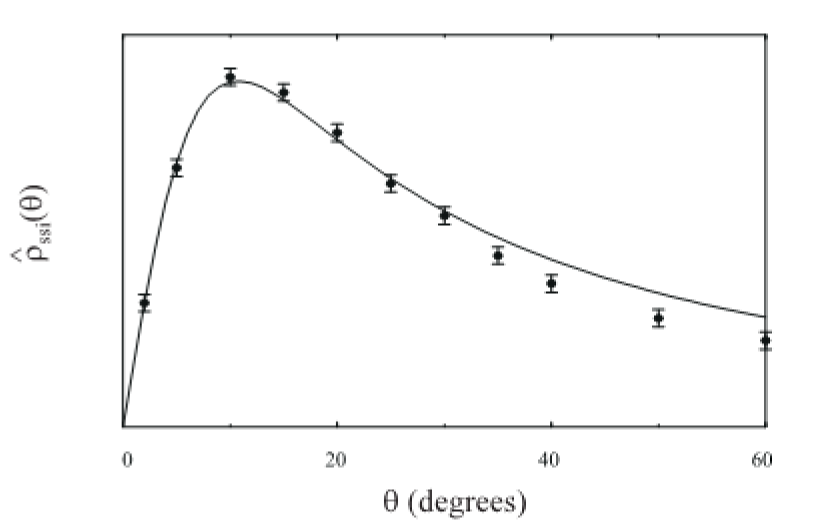
$$
\begin{aligned}
\rho_1 &= \rho_0 e^{-T_{E_1}/T_2}\left(1 - e^{-T_R/T_1}\right) \\
\rho_2 &= \rho_0 e^{-T_{E_2}/T_2}\left(1 - e^{-T_R/T_1}\right)
\end{aligned}
$$

$$
\hat{T}_2 = \frac{\left(T_{E_2} - T_{E_1}\right)}{\ln\left(\frac{\rho_1}{\rho_2}\right)}
$$

$$
\frac{\sigma_{\hat{T}_2}}{T_2} = \frac{\sigma}{\rho_0} \cdot \frac{T_2}{\left(T_{E_2} - T_{E_1}\right)} \cdot \left[\frac{\sqrt{\left(e^{-2T_{E_1}/T_2} + e^{-2T_{E_2}/T_2}\right)}}{e^{-\left(T_{E_1}+T_{E_2}\right)/T_2}}\right]
$$

$$\hat{\rho}_{\mathrm{ssi}}(\theta) = \rho_0 e^{-T_E/T_2^*} \frac{(1 - E_1)\sin\theta}{(1 - E_1\cos\theta)}$$

$$\frac{\hat{\rho}_{\mathrm{ssi}}(\theta)}{\sin\theta} = E_1 \frac{\hat{\rho}_{\mathrm{ssi}}(\theta)}{\tan\theta} + \rho_0 e^{-T_E/T_2^*}(1 - E_1)$$



(a)